“A CAPACITY FOR THE SUBLIME”: MATH AND ART AS EXPERIENCE

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Abstract. We present a rough classification of math-art pieces, based mainly on the relationship between math and art that they employ and demonstrate, as well as their subsequent functionalities. Hermann Hesse's 1946 novel *The Glass Bead Game* is invoked to clarify and illustrate our classification. We analyze a range of math-art examples through this lens, and, in particular, describe several math-art pieces that we the authors have published or exhibited over the last five years of both individual practice and collaboration.

1. Introduction

> “The mathematicians brought the Game to a high degree of flexibility and capacity for the sublime, so that it began to acquire something approaching a consciousness of itself and its possibilities.” — The Glass Bead Game [5, p.32]

Let the term *math-art* refer to any work that intentionally engages both math and art. Our goal in this paper is to present a vision of what math-art can be and an expansion of our expectations of the cross-disciplinarity. We will present a rough classification of math-art pieces, based mainly on the relationship between math and art that they employ and demonstrate. Our examples of math-art are drawn from several sources: the annual Bridges conference, the Journal of Mathematics and the Arts, the art history lexicon, our own collaborations, and popular culture. The classification, described using levels, is not meant to be rigid or exhaustive; many pieces fit into multiple levels and some pieces are unclassifiable in our system. We only aim to articulate several broad categories that emerge when one asks, “What is the relationship between math and art in this work?” and use these categories to — while honoring the work that has been done — point to new directions and possibilities for math-art.

The paper is organized as a simple progression through the levels. First we discuss Level 0 work, which is mainly focused on identifying mathematical aspects of art pieces, or artistic aspects of mathematical entities. Math and art relate via content illustration. In Level 1 work, math informs the generative process and aesthetics apply selective pressure; the result is usually pieces that are mathematically interesting and visually appealing. Math and art relate through edited representation. In Level 2 work, math and art relate through structure. The math evoked is more abstract and the art manifests more conceptually; these pieces are less representational and enfold material, organization, and context in their intentioned

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outcome. Finally, Level 3 sees math and art as two manifestations of a certain way of processing, and perhaps understanding, existence. The pieces are often time-based, experiential, ontological, and transcend but include both math and art; they may not be recognizable as either.

We analyze a range of math-art examples through this lens, and, in particular, describe several math-art pieces that we the authors have published or exhibited over the last five years of both individual practice and collaboration.

Our classification scheme is inspired by Hermann Hesse's 1946 novel *The Glass Bead Game*. As we will show, the concept of a Glass Bead Game helps to clarify and illustrate math-art work of Levels 2 and 3, while distinguishing them from work at Levels 0 and 1.

2. Prologue: *The Possibility of Walking Through a Wall*

Before we introduce our classification scheme, we give one example of a math-art piece we the authors created in 2007. *The Possibility of Walking Through a Wall* is an installation including a projected video on loop which depicts, in profile, the artist's ceaselessly repetitive attempt at walking through a wall. Alongside the projection hang several pages of scratchwork, drafted by the mathematician in his attempt to calculate the artist's quantum mechanical probability of walking through the wall in question over a prolonged time-span. These calculations aim to be a prescription for the artist's behavior, her theoretically precise odds of success for tunneling through the wall over a given period of repeated attempts customized using her and the wall's mass, volume, and composition. The complexity of the problem and propagation of approximations were prohibitive in the endeavor; the computation was abandoned and never fully completed. Indicated on the pages, the mathematical model employed conceptually mirrors the artist-wall system. Taken together, the scratchwork and video point to a convergence of the mathematician's experience and the artist's experience as the same effort in the face of frustration and hopelessness. The first version of this piece was shown at *Current Gallery* in Baltimore in 2007, and a second version is anticipated to be exhibited at *Toves Galleri* in Copenhagen in 2013.

3. Level Zero

Definition 3.1. At Level 0, math is translated directly into art, or art is translated into math, with little-to-no further processing or conceptualization. It may also be the case that a work is the result of some mathematical element being recognized and extracted from an art work, or vice versa. In these cases, a mathematical or artistic conclusion is being drawn after the fact, after the prior existence of the other.

The translation between math and art goes in both directions. In one direction, for example, much fun can be had analyzing the geometry and symmetries found in Islamic patterning of tiles, screens, and stonework. Math, as a study of pattern, can be applied to a wide range of historical artworks. While drawing a connection between math and art, the fact that this is often done using anachronistic mathematics also points to the fundamental distance between the math and the art here.

As with any act of translation, the differences of worldview are as prominently underscored as any commonalities of content. As another example, statistical analysis of paintings allows us to quantify and appreciate the uniqueness of the artist, and in some cases distinguish the hand of a master from a counterfeit.

In the other direction, most math-art works involving fractals classify as Level 0. The mathematical object is simply put on a wall and called art. Elementary mathematical poetry does the same, by displaying the exoticism of various mathematical terminologies and phrasing.

In recent decades, mathematics has appeared in works of theater, popular cinema, and television — e.g. *Proof*, *Pi*, *A Beautiful Mind*, *Numb3rs*. Again, the content and practice of mathematics is simply translated into the new medium and displayed, albeit with the dramatic and hyperbolic manipulations common to these forms.

Level 0 math-art is common, and draws interesting connections between the two disciplines of mathematics and art. The result of experiencing such a piece is usually a feeling of *appreciation*; the experiencer has a welcomed excuse to examine and go deeper into the work. But these pieces struggle to invoke both contemporary mathematics and contemporary art, and rarely present a surprise or singular experience. There was a time, however, when what we are now calling Level 0 math-art work was revolutionary. In the early 1960s, Sol LeWitt, considered one of the founders of Minimalism and Conceptual Art, used the simplicity and modular potential of the cube in order to build his sculptures, which he termed "structures". The cube is a form both modernist and postmodernist artists have returned to time and again in their interrogations of conventional authorship, representation, and objects as they relate to the human body and perception. Earlier on, Donald Judd advanced his slick, machined steel cubes as an echo of the products of industrial mass production and in an attempt to erase the artist's hand from the art object. In this sense, stark geometry was employed as a device for attempting to attain "objective" forms — forms that existed outside of the artist's mind and hand and that were extricated from the space of expression and pictorial representation; this exploration of Platonic space very much overlaps that of mathematical ontology.

Of course, post-structuralist developments, such as existential phenomenology, have since flooded artistic practice to upend the original aims of Modernism. As modernist questions transitioned into the space of Postmodernism, artists continued to grapple with notions of universal form, from the work of Judd and Lewitt on to Smithson. Robert Smithson's *Spiral Jetty* is an earthwork for which the artist laid mud, stone, and water in the form of a large spiral that jutted out from the Great Salt Lake shore and was barely exposed at the surface of the water, its visibility dependent on the variable water levels. From the same generation, the French artist Bernar Venet, who is heavily influenced by LeWitt, extracts formulae from mathematics texts and presents them as larger-than-life wall drawings [6]. The commutative diagrams make reference to contemporary mathematics, but the relationship between this math and art, as it functions in the 21st century art context, remains Level 0.

4. Level One

Definition 4.1. Level 1 sees math as a generative medium for artworks, treated as another material, just as steel or wood or paint — for example, the use of some mathematical algorithm to determine a process that generates a work. This can proliferate a lot of material, and one commonly uses some artistic criteria to edit and select the most compelling outcome(s).

A lot of "mathematically-inspired artworks" (for example, on view at the Bridges art exhibition) is at this level. When presented with such a piece, one is often hit with the feeling that *something mathematical is going on*, something that would perhaps be interesting, once one knows what it is. Whether or not we find out, the meaning of the piece is in its relationship to the mathematics "behind" it.

Math-art sculpture embodies this feeling well. Necessarily geometric, the power of such a piece comes in part from its invocation of mathematics, even basic mathematics[1]. Recently, the growth of three-dimensional printing technology has given math-artists a straightforward way to represent interesting and more elaborate mathematics. For the time being, the current dependence on less-than-enchanting pale white plastic for printing medium almost forces the viewer to avoid contemplating materiality or presence, and instead withdraw to the pre-manifested realm of mathematical generative principles. When working with these emerging representational technologies (whether 3D printers or software), it is important to consider that, as with all vestiges of our material culture, someday this new material, too, will date itself and become imbued with its own set of historical references, deneutralizing it. One must always observe these new materials and methods with the critical distance of an inevitable future and own the fleeting conditions of newness so the effect of the artwork, as it ages, does not become dominated by its own obsolescence. To quote the designer and critic Ellen Lupton: "Be careful, your technology is showing".

The vast genres of algorithmic art and generative art, commonly understood to involve computer code at some step of the creative process, fall squarely in Level 1. In the most common case, some algorithm, implemented on a computer, creates images that then must be sifted and evaluated artistically. This tried-and-true recipe generates work that can be truly mathematically interesting and visually appealing. As discussed in a recent Special Issue of the Journal of Mathematics and the Arts, the artistic judgement can even be computerized to some extent.

Music that involves mathematics also usually follows the Level 1 recipe. At least since Hanne Darboven in the 1980s, and arguably beginning with John Cage in the 1960s, there is a long tradition of algorithmic music, ranging from the sound of the digits of π to new musical scales based on logarithms.

Besides using algorithms, math concepts, or equations as generative input, one can use mathematical practice itself. The Miami-based artist Lun-Yi Tsai captures contemporary mathematics in an informed way in his drawings and paintings, using paired collaborations with academic mathematicians. His collaborations involve an iterated process of conversations, mathematical study, attempts at visualizing the mathematics on paper or canvas, and feedback.

[1] And conversely, mathematics can invoke sculpture: "Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, like that of sculpture". — Bertrand Russell

The Modernist movement being so very formative and recognizable to Western culture, many contemporary math-artworks understandably harken back to the forms and discourses of Modernism, the goals of which, over the course of decades in the art lineage, have been absorbed by the myriad movements of Postmodernism and Conceptual Art that have emerged from the 1960s to present. For this reason, *math that looks like art usually looks like modern art*, which causes it to function more as an inert, art-historical illustration or description than an actively representational work that can be posited in the contemporary progression of artistic practice. The strongest Level 1 pieces address the subtle but important distinction between approaches to visualization, for instance, description and simulation versus representation and making. A description refers to something in the real world and to real objects. By contrast, simulation is much more hypothetical and less empirical. A representation lies somewhere between empirical descriptions and hypothetical simulations; it can represent abstract things, and it does not need to be precise. The artist's practice is this intuitive synthesis which results in representation.

Finally, we must admit that the distinction between Level 0 and Level 1 is not always apparent or even well-defined. We have seen papers analyzing the group theory found in English line dancing; this is Level 0. We have also seen video pieces using Hungarian line dancing to demonstrate various sorting algorithms; this is Level 1. When presented with a three-dimensional sculpture of a knot, it is not immediately clear whether it is simply an enhanced representation of a mathematical object, hence Level 0, or whether some non-trivial mathematics determined this particular sculpture's construction, thus Level 1. Perhaps the answer depends on the impossible task of tracking the non-rational creative impulses and mathematical intuition of the artist.

5. Level Two

Definition 5.1. In Level 2 math-art work, the role of mathematics in the piece takes on an organizational and conceptual aspect, and the art is more idea-based. The two components mutually reflect each other, rather than one simply determining the other. Mathematics informs the organization and structure of the piece, rather than just appearing as the content or impulse of the piece. Consequently, the piece is experienced in a mental rather than purely audio-visual space, in which abstract theoretical/meta concepts can be explored. Higher meaning is generated by this feedback system. Level 2 includes works that begin to collapse the binary of math and art even if not entirely or evenly. The medium becomes part of the message, and focus shifts to ideas.

We will give several examples of such pieces, then discuss idea-based art works and the art world. Following that, we will summon Hesse's *The Glass Bead Game* to further illustrate this level and contrast it with Levels 0 and 1.

Example 5.2. *Imagining Negative-Dimensional Space.*

At the Bridges 2012 conference on mathematics and art, we presented a 90-minute workshop entitled *Imagining Negative-Dimensional Space* [9]. The goal of the workshop was "to induce the experience of contemplating negative-dimensional space". The workshop had four phases. First, we lectured about space and dimensions. Second, alternating with guided meditations to clear the mind, we led a variety of frustrating thought experiments attempting to visualize negative-dimensional space, only to conclude that our visual metaphors were useless. Third, the participants conducted a ceremonial circle dance, meant to bypass their rational minds and induce non-rational insight and a negative-dimensional space experience. Finally, there was discussion and follow-up questions.

Although this was not explained to the participants, the organization of the workshop was mathematical. It was based on Jacques Hadamard's 4-stage theory of mathematical discovery [4]. Hadamard argues that the first stage to any mathematical breakthrough is diligent work and preparation. The second stage is a confused and frustrating stuckness. Then, after some relaxation the insight may emerge in a sudden, certain, non-linear burst of realization. The fourth stage is mere verification and documentation.

This piece was Level 2 because it contained mathematics in its content as well as its structure. Whether or not participants were familiar with Hadamard, the piece was engaging and commenting on his theory, on the subjectivity of experience, and on the robustness of the mathematical process.

Example 5.3. Nathan Selikoff's *Untiled Faces*.

A recent example of a successful Level 2 piece is Nathan Selikoff's *Untiled Faces*, which won the award for Most Innovative Work at the Bridges 2012 art exhibition. The piece is an interactive device with three small screens and three small joysticks that allow the experiencer to navigate the four-dimensional parameter space that determines a certain strange attractor, as well as manipulate the perspective on a visualization of the attractor at each set of coordinates.

More than just presenting a visualization of one specific 2D slice of a strange attractor, as many Level 0 and Level 1 math-art pieces have done in the past, Selikoff's piece encompasses the entire four-dimensional object. The physical organization of the piece — the multiple screens and joysticks in serial, with their elegant and simple design — accentuate the 2+2=4 dimensions that can be explored, and the two degrees of freedom one has with an equidistant perspective on the object. And more than simply enjoying one static image, the experiencer is invited to explore and experience this *dynamical* system. The piece becomes not about a mathematical object, but the multiple, changing perspectives required to see it in all its richness.

Example 5.4. Wolcott's Ph.D. dissertation.

The second author's mathematics Ph.D. dissertation [10] is presented as a Level 2 math-art piece. The chapters are organized using the structure of a 1-simplex and its suspension. A 1-simplex is a triangle: three vertices and lines connecting them. The suspension adds a point above and below the plane of the triangle and connects every point of the triangle to the two points. The result, to a topologist, is a two-dimensional sphere.

Chapter 1 in the dissertation is an Introduction, explaining the organization. Chapters 2, 4, and 6 are full of rigorous mathematics — definitions, theorems, and proofs. Taken together these even-numbered chapters, the 1-simplex, are complete and sufficient. Chapters 3, 5, and 7 are meta-mathematical complements to the even chapters, discussing aspects of the lived mathematical experience and community of math practitioners. As explained in the Introduction, paired Chapters 2/3, 4/5, and 6/7 are to be located in Thailand, Siberia, and Seattle, respectively. Choosing to engage both the rigorous and meta-mathematical results, and thereby adding a dimension — the human dimension — means taking the suspension of the 1-simplex. Topologically, this results not in a bare and cold triangle, but in a 2-sphere, the surface of the globe.

In this example, the non-trivial mathematics of simplices, suspension, and topological equivalence provides a poetic organizational framework for the chapters of the dissertation. The reader, with the image of the triangle and the globe, is left to contemplate the relationship between rigorous, conventionally written mathematics, and self-aware reflection on the mathematical experience and community.

5.1. **On Institutional Similarities Between the Worlds of Math and Art.** While many may be quick to point out the differences between math and art, at the professional level, there are many institutional similarities between them as well as parallels in their respective codifications. In the art sphere, there are well-defined terms, there is recurring vocabulary, and there is a precise and cumulative exploration of relationships between these conceptual entities. Let's take Duchamp's work *Fountain* as an example of a widely accepted piece of "material vocabulary": urinal = Duchamp. If one makes a work of art using a urinal without intentionally referencing this ingrained fact of art history, it is comparable to ignorantly declaring "1+1=3". If there were a definition, one might say mathematics is the entirety of knowledge that has been systematically and consistently built using necessary logic and some fundamental axioms about number, space, and process. In the same way, one might define the "high" art world to be precisely the community in which artworks are necessarily embedded in history and for whom art practice is a *consistent*, and continually expanding, exploration of certain fundamental themes.

Of course, one must concede that there is a seam wherein math and art start to pull away from each other. Mathematical themes are limited to the above-mentioned number, space, and process, whereas the universal themes of art are more diverse — perhaps even enveloping everything. And while math emphasizes rigor and logic in building its castle, art emphasizes relevance and resonance. However, both are established and consistent systems of thought and action.

Furthermore, just as in the math regime, there are also well-established standards of academic and technical rigor in the art world, and the weight of each depends on the medium with which the artist works (e.g. paint, video, concepts) and the artist's place in history (are they working as a modernist sculptor in 1953 or in 2013?). Within the institution of art — which is the ultimate determiner of the allocation of funding, resources, and publication, thereby dictating which works are realized and entered into the art-historic stream — artists and artworks are the subjects of intense scrutiny. Analysis, scepticism, and incredulity are applied, not only by gallerists and collectors, but also by theoreticians, philosophers, writers on culture, and book and magazine editors. To have an artwork considered critically and to make a contribution towards new ways of engaging the cumulative artistic language, the artist must have enough of a command of that language to break free from its restrictions, opening it up to new forms — that is, other possible futures of human capacity and creation.

5.2. **On Glass Bead Games.** We believe the concept of Level 2 (and 3) math-art is captured by the notion of a Glass Bead Game (GBG). Set in some undated future version of our world, Hermann Hesse's *The Glass Bead Game* [5] imagines an intellectual culture that perfectly balances depth and specialty with breadth and synthesis. Knowledge and education are entrusted to The Order and their system of schools, supported by society at large but removed from its politics and banalities. Members of The Order, lifelong scholars and teachers, study all disciplines and specialize in more than one. Within The Order is an elite group of polymaths, experts of everything, that orchestrate intricate manifestations of knowledge called Glass Bead Games.

> "The GBG is thus a mode of playing with the total contents and values of our culture; it plays with them as, say, in the great age of the arts a painter might have played with the colors on his palette. All the insights, noble thoughts, and works of art that the human race has produced in its creative eras, all that subsequent periods of scholarly study have reduced to concepts and converted into intellectual property — on all this immense body of intellectual values the GBG player plays like the organist on an organ" [5, p.15].

In practice, a GBG is part performance, part lecture. A universal language has been developed, derived mainly from musical and mathematical notation. As a sort of calligraphy, it allows for the expression, juxtaposition, and manipulation of any ideas from any discipline. The beauty of the Game is thus not in its representation, as much as in its structure and contents, and the interaction between structure and content.

The organizational structure of a Game might be drawn from music; a prominent example in the novel describes how two contrasting ideas are developed independently, then intertwined like two voices in counterpoint. Another example is structured on modern classical music: with ideas and concepts as musical tones, their "harmonization underwent a whole series of refractions, of splintering into overtones, and paused each time, as if wearied and despairing, just on the point of dissolution, finally fading out in questioning and doubt" [5, p.148]. Or it may come from architecture; for another Game described in the novel, the "idea was to base the structure and dimensions of the Game on the ancient ritual Confucian pattern for the building of a Chinese house: orientation by the points of the compass, the gates, the spirit wall, the relationships and functions of buildings and courtyards, their coordination with the constellations, the calendar and family life, and the symbolism and stylistic principles of the garden" [5, p.245]. It is within this framework that the various themes of the Game, whether mathematical, musical, scientific, or theological, would be articulated and explored.

A GBG-inspired work of math-art, then, would first and foremost be based on ideas. The real medium is ideas. It would also be self-aware in a sense; the organization of the ideas would have a nontrivial, mathematical structure that informed and was informed by the content. Not only is the overall conceptual organization part of the work, but every aspect of the work must resonate and be justified. In particular, a GBG math-art piece would be intentional about its physical medium, whether paint, sculpture, video, or performance. It would be aware of its place in the world of math-art, and reference its relationship to other math-art works that have been done.

The role of mathematics in such a piece would necessarily be more abstract, but this presents only opportunity, as the 20th and 21st centuries have seen an explosion of abstraction in mathematics. Level 2 gives us more opportunity for introducing truly contemporary mathematics into math-art work.

More broadly, we underscore that the structure of "the game" is a powerful one, having also been exercised often in artistic approaches in the last century, most notably by the Situationist International of the 1950s and 60s. French philosopher Guy Debord's *Society of the Spectacle* [2] was an influential work of the 1960s which, while heavily critiquing the realm of entertainment and illusions of autonomy from the mass market, also recognized the path out of the passivity of the spectacle as the free activity of the game, or more radically, *détournement*. Excellent examples of this in practice were the absurd and playful, though profoundly critical, actions of the Situationists. This kind of approach to artistic activity utilizes the hegemonic conditions of life in order to rupture and overcome them.

Before moving to the next level, we wish to elaborate on the difference between Levels 0 and 1 and Level 2. In Hesse's novel, the GBG is described as developing historically out of the "Age of Feuilletonism", which was marked by superficiality and empty virtuosity. In this era, there was a proliferation of intellectually shallow, usually non-rigorous, and in the end meaningless, dabbling. (Think crossword puzzles and news show talking heads.) "The life of the mind in the Age of the Feuilleton might be compared to a degenerate plant which was squandering its strength in excessive vegetative growth, and the subsequent corrections to pruning the plant back to the roots" [5, p.33]. The Order, and its GBG, brought depth, rigor, and true interdisciplinarity to our search for understanding.

There is certainly a tendency, in many recent math-art pieces of Levels 0 and 1, to aestheticize math that is not very deep (e.g. fractals, symmetry) and present it in a form that is artistically ignorant (e.g. of previous work, of the inherent meaning of materials, even of the color wheel). These pieces have a place in mathematics, and a place in art. But mathematics is more than recreational math, and art is more than "craft". Mathematics and art, to be alive and pertinent, must continually strive for more depth, new insights, larger questions, richer connections, and ultimately for transcendence.

6. Level Three

Definition 6.1. At Level 3, the art and math both operate at a sophisticated level for their respective fields of expertise. The math-artwork is consciously posited in not only the math-historical, but also the art-historical stream and the cultural discourse, and there is a simultaneity of the development of the two elements with both artistic and mathematical intentionality inherent to the making. The math and art cease to be discrete components, and their link in the math-art piece becomes inextricable, transcending the sum of their parts in a new indivisibility; the work does not fall into binary didacticism. The mathematician and artist cease to be mere technicians in their respective fields and create a work that, while including technical rigor on both fronts, enters the work into the ontological realm — the Meta.

An effective bonding agent of cultural activities is theoretical space, which gleans from all fields in order to create a lens through which one can view the world, operating as an umbrella under which all of our intellectual, creative, libidinal, ludic, and prosaic actions and processes can hold hands, informing and reflecting each other. For decades, art has been intimately and assumptively intertwined with linguistics, philosophy, the culture industry, etc. Mathematics is not exempt from this integrative milieu. If math at all informs or is a result of the impulses of the human condition, then it also has its place in (the language and forms of) art. At Level 3, the articulations of the math-artwork emerge from the complex and precise language developed by the history of intellectual, visual, and material culture and synthesizes these elements into something that, rather than functioning as a simulacrum, functions to provide a new perspective and reframe what is already here, so we may imagine what may be.

In *The Glass Bead Game*, Hesse actually distinguishes between two types of Game, the "formal" and the "psychological".

> "In the formal Game, the player sought to compose out of the objective content of every game, out of the mathematical, linguistic, musical, and other elements, as dense, coherent, and formally perfect a unity and harmony as possible. In the psychological Game, on the other hand, the object was to create unity and harmony, cosmic roundedness and perfection, not so much in the choice, arrangement, interweaving, association, and contrast of the contents as in the meditation which followed every stage of the Game. All the stress was placed on this meditation. Such a psychological Game did not display perfection to the outward eye. Rather, it guided the player, by means of its succession of the precisely prescribed meditations, toward experiencing perfection" [5, p.197].

What we have described as a Level 2 math-art work would be considered a formal GBG. Level 3 math-art works, then, are psychological GBGs, if for "meditation" in the above quote we instead read "experience". These works embrace time and interiority as parameters. They see math and art as two reflections of the same thing: *an intentioned movement towards authentic experience, the creation of a make-believe space to induce a feeling of understanding.* If there is truly meaningful common ground to be found between math and art, it is in experience, doing, and process, not content alone.

Example 6.2. *Imagining Negative-Dimensional Space*, revisited.

> "Whereas the beautiful is limited, the sublime is limitless, so that the mind in the presence of the sublime, attempting to imagine what it cannot, has pain in the failure but pleasure in contemplating the immensity of the attempt." — Immanuel Kant, *Critique of Pure Reason* [7]

This performance/lecture workshop, described earlier, problematizes existence and knowledge. Negative-dimensional spaces, special cases of “spectra”, exist in the minds of the stable homotopy theorists who discuss and prove theorems about them. Negative-dimensional space cannot be “seen”, but through working with spectra, one develops an intuition of how such “spaces” work. Can this understanding be communicated to a non-mathematician? Can the experience of negative-dimensional space be glimpsed or induced in a novice? The object of this math-art piece is the possibly-unknowable object of negative-dimensional space, or rather the lived experience — the excitement, effort, frustration, and perhaps the non-rational singular taste — of contemplating such an object.

This ambiguity is further accentuated, as the participants are informed that they are in a workshop, a preparatory experiment for a future performance. They are thereby forced to continually self-reflect, and evaluate their ongoing workshop experience through the lens of imagined future minds. But there is no future performance; the workshop is the piece. The participants think that they are “extras” in the spectacle [1], but in fact they are the protagonists, they are the piece.

Objectless and overhyped, the participants are left with nothing but the truth of their experience. What began as a conventional mathematics lecture, transitioned to a guided meditation through thought experiments, and concluded with ceremonial drumming and hypnotic circular dance, has taken them on an internal and collective journey through the emotional and cognitive landscape of mathematical experience. Whether or not they realize it, they have in fact experienced the contemplation of negative-dimensional space. The impossible goal of the workshop was attained.

Example 6.3. *A Tree Calls.*

This action took place on April 15, 2012, in the woods of Washington state, USA, in Copenhagen, DK, and everywhere along the great circle connecting these two points. The project primarily manifested as a performance-action via a walking tour, but also included art objects, e.g. hand-painted and printed maps, wall text, relics, sound, etc., which supplied multiple points of entry for both viewers who had joined the walk and had not. This “walk to meet a sound” was physically bracketed by the artist and her audience in Copenhagen and the mathematician and his tree in the forest; it was conceptually bracketed by their respective rigors of practice.

The work was first presented as a printed announcement and invitation for the performance:

> A TREE CALLS
> a walk to meet a sound, 15 April 2012
>
> A tree will fall in Mount Baker National Forest in Washington State, USA, on Sunday, 15 April 2012 at 00:00:00 United States Pacific Standard Time. The sound of the tree hitting the ground will take 6 hours, 31 minutes, and 47 seconds to travel 7686.35km. It will arrive in central Copenhagen on 15 April at 15:31:47 Central European Summer Time. You are invited to join us in a walk

> to meet the sound. This walk will begin at Toves Galleri on Sunday, 15 April at exactly 09:00:00 — the moment the tree hits the ground in the faraway forest — and will last for the time it takes the sound to travel from the forest to us. Please be prepared to leave your phone and internet devices behind, and to enter the void.

To accurately calculate the time it would take for the sound to travel from Washington to Copenhagen, the great circle was divided into 20 equal segments. Extensive meteorological research was done to assess the probable local air temperature at the appropriate time on the day of the event, and this was used to calculate the local speed of sound. The mathematical great circle, with the computations made at intervals along its distance, provides a framework within which to hang ideas of faith, incredulity, the absolute versus the real, precision and truth, and absurdity. So it is Level 2, but, at the same time, we use mathematics in a Level 1 way — the speed-of-sound calculation determined when the piece was to happen in Copenhagen.

On one hand, *A Tree Calls* could occupy Level 2 since the static great circle and its map help to "sustain a mental space" in which to consider rigor, imagination, faith, distance. But the piece can then be bumped up to Level 3, in that these ideas are ideas about experience, and mathematical experience at that. Does it contain non-trivial mathematics, and contribute to mathematics? Any new perspective on the role and use of rigor is a new perspective on math. The central role of the object as the sound, physically negligible but conceptually undeterred as it makes its journey, is a graceful way of sliding us away from the thing and towards the idea of the thing, which is what math is made of. The sound, when it arrives, *is* a mathematical object. Perhaps if any math-art work is going "to contribute to mathematics", it is going to do so by saying something about, or giving a new perspective on, mathematical *practice*.

Example 6.4. *The Possibility of Walking Through a Wall*, revisited.

This piece, introduced in the Prologue, also operates at several levels. The mathematical scratchwork on the wall is presented as such (Level 0); the probability calculation it contains is a prescription for how long the piece should continue (Level 1); the model used in the calculation conceptually parallels the artist-wall system (Level 2).

But further, *The Possibility of Walking Through a Wall* is a work that utilizes laboratory-esque trial and error, visual representation and repetition, and analytic computation to get to the heart of a concept that rises above both visuality and calculation: the human condition of unknowing. As a Level 3 work, this piece simultaneously wields and dismantles the "authority" assumed of both artistic vision and mathematical certainty. When confronted with this insurmountable task exercised in real time (i.e. in the limited endurance of live performance, in the video loop *ad infinitum*, in the hand to ink to paper accumulation), at first we think we can retreat into the certainty of calculation — calculation always brings us closer to knowing more, right? A collapse of this promise quickly ensues, the mathematician and the artist both sliding down their separate slopes to meet each other. And in this slide towards the imminent failure of knowing, the point of this work is not to arrive at a definitive top, bottom, or end. Rather, the work resides in a process bracketed by the untouchable limits of ultimate knowing and ultimate not knowing; this limitlessness in the face of limitation is the sublime.

The evolution of this piece and its meaning is captured well in an excerpted email exchange between the artist and the mathematician, which we have included in an Appendix.

This work navigates the tension between probability and possibility, employing the architecture of probability in pursuit of affirmation of possibility, until, eventually, the very attempt to assess that probability drifts ever further away from being itself possible. With this departure from the possibility of knowing probability, the attempt to walk through a wall becomes an act of faith in possibility, absent of the determination of probability. Each instant before each inevitable collision is momentarily pregnant with the potential for the birth of a moment in time when matter and space align to collapse into each other, annihilating subject and object in an interval of unrelenting agitation. If Deleuze and Guattari's "war machine" [3] is the nomadic, decentered and uncoded opposition to the static, centered, coded State — by definition, kinetic yet arrested in its potentiality and destined never to arrive — then perhaps the cleft between the rational known and the (non-rational) dream of the unknown is where this war machine lives. And if this nomad is characterized by its absolute exteriority, then what would it mean to actually succeed in walking through the wall? What happens when the exterior becomes interiorized, the uncoded absorbed and colonized by the coded, multiplicities of the future overthrown by certainty of the present?

Example 6.5. Gordon Matta-Clark's *Conical Intersect*.

Gordon Matta-Clark is known for the seminal, site-specific work he made in the 1970s, appropriating abandoned buildings that were marked for demolition to make way for urban and suburban renewal projects. He called this series of architectural interventions "building cuts", wherein he would gore and remove sections of the buildings, using the violence of handheld demolition tools to create a delicate sliver down the center of a building or a soft round hole through its middle. The building is left a quiet, vulnerable body, cut open with dark corners illuminated and in duet with the light and dark play of the moving sun until finally extinguished once and for all by the city's wrecking ball and bulldozer.

For *Conical Intersect*, Matta-Clark obtained temporary access to an old building slated for demolition next to the construction site of the Centre Pompidou — a highly problematic intersection of gentrification, French history, socio-economic tensions, and the industrial propulsion towards commercial progress. He carved a giant cone into the building. When the cut was complete, the work stood sentinel, silently awaiting its fate while reflecting its environmental context, the frozen object a jarring interruption in a landscape of flux.

This piece is a prime example of Level 3 work because it carries a fundamental geometric form into architectural, phenomenological, socio-economic, and political space, nodding to mathematics and Minimalism while evoking the properties of the vernacular telescope as it relates to human vision, both immediate and historical. In the simplicity of this ambitious intervention, there are many levels at work. First, we can approach the piece from a formal perspective. The mathematics in *Conical Intersect* is remarkably rich and elegant. By intersecting a cone with a plane at varying angles, one recovers all the conics — circle, ellipse, parabola, hyperbola

— nothing more and nothing less. These "conic sections" have been studied at least since Apollonius of Perga in 200 BC, and have provided a beautiful context in which to investigate the interplay of geometry, curves, and algebraic equations, even up to present-day modern algebraic geometry. The simple act of removing a cone from an average building, gridded with floors and walls, empties a space while filling it visually with diverse demolition curves. (The mathematician needs to know: is that a parabola or a hyperbola?) Rather than a plane intersecting a stable cone, the agency of the conventional mathematical construction is inverted, as the cone intersects the building in a *de*struction.

Meanwhile, this cone is a device that, at its overwhelming architectural scale, guides the line of sight of passersby through the raw, decrepit building interior and into the young body of the new cultural center. In her essay *On the Holes of History* [8], Pamela Lee extensively examines the layers of this work, revealing sediments of a lengthy national narrative as well as the flexing accumulations of the rapid advancement of the mid-twentieth century — progress as an insatiable mining drill. Would it emerge into a new light or thrust continuously into darkness?

The title *Conical Intersect* encompasses so many intersections that speak to math and art as well as transcend them: those of colliding geometric forms, positive and negative space, darkness and light, history and future, the protracted moment and unyielding acceleration, the phenomenological and the sublime. All this is accomplished by the simple mathematical act of carving out a cone. This is an example of how a basic interaction of mathematical forms has the capacity to deeply resonate with the human condition.

7. Conclusion

There is a certain necessity to math-art. Once we decide to use a 3D printer, and decide the geometric formula we want to express, the object is determined. In the idea-based pieces like *A Tree Calls*, the requisite of elegance and simplicity fates the result — once the ideas are arranged, the piece unfolds necessarily.

The same necessity arises in mathematics research. This gives the feeling of discovery: it had to be this way. However, *we may discover results, but we create perspectives.* The true genius of mathematics comes from knowing which questions to ask, what definitions to make, where to look. This amounts to creating a new perspective on a poorly understood region of the mathematical landscape.

The purpose of this paper has been to map the maps of math-art. We have attempted to describe the common perspectives on math-art, Levels 0 and 1, and draw attention to the potentialities of the more subtle Level 2 and 3 perspectives. Each of these levels presents a vision of the relationship between mathematics and art; each is a valid and worthy vision that inspires powerful math-artworks. Our hope is that the math-art community can continue to expand and embrace new perspectives, transcend and include, build up from deep roots and grow with vitality and true contemporaneity.

8. Appendix: Excerpted email exchange about *The Possibility of Walking Through a Wall*

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> From: Luke Wolcott
> Date: Apr 5, 2007 11:47 AM
> Subject: the wall and stuff
> To: Elizabeth McTernan
>
>
> So last night I had an idea. You can tell me this is the wrong
> direction to take it, but I thought your art was sort of a "let's try
> it and see what happens, and what happens is the art". There is a
> chance I could solve this problem. There's a chance I could get to
> the other side, and give a number that was close to the right answer.
> But I can't give the exact answer, because there are too many
> variables and at this stage no one knows if there are ways to simplify
> the problem. Even making a lot of (mostly justifiable)
> approximations, there's (triple) integrals like that one to work out.
> There's a metaphor between you banging against the wall, and me
> banging against this (nearly) impossible mathematical barrier. The
> math solution is not going to be elegant or short, it's going to be
> arrived at through a long tedious persistence of approximating and
> smacking theory/formulas against reality. As you continue banging
> against the wall, on and on, your hope and idealism will get
> extinguished even as your chance of getting to the other side is
> steadily improving. As I work along on this problem, my faith in the
> answer I might get being close gets smaller and smaller, even as my
> chance of finally getting some approximate answer is increasing.
>
> In an absolute sense, we think that to all the questions we can ask,
> we can figure out an answer. While that may be true, some questions
> can't be answered in a hundred lifetimes of time, or with all the
> knowledge/techniques us humans have developed. The idea of banging
> against a wall until you teleport through seems hopeless, the idea of
> figuring out the math problem of "what is the chance of that?" seems a
> little less hopeless, but then there's the idea of searching for the
> questions "who am I, what should I do next", which are equally
> hopeless. [You and I are both] looking for answers to (nearly)
> impossible questions, and we have no way of knowing if there's a
> realistic chance of getting to the other side. The question is hard,
> but the question "can I even find an answer?" is also (nearly)
> unsolvable. So we keep going... If I told you unequivocally
> that it'll take 10^50 years until you have even a decent chance of
> making it through, would you persist? If I knew for sure that by the
> time I worked out an answer to the problem, I could be off by a factor
> of 10^20 years, would I even bother?
>
> Maybe it's easier for you to teleport through than it is for the
> physicists to figure out the chances of you teleporting through.
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> Sometimes I think it's easier to know the answer than it is to figure
> out how you might figure out the answer. Like who am i?
>
> Anyway, I don't know what direction you want to go, but playing the
> video with a little note saying "i'd have to do this for 4.3 x 10^50
> years until I had a 1/10th chance of making it through" doesn't seem
> as interesting as playing the video next to my frustrated scribbles
> and equations and saying "we have no idea how long we'd have to
> continue until we have a chance of even knowing what our chances are".
>
> Let me know what you think.
> Luke
>
>
>

****
From: Elizabeth McTernan
Date: Sun, Apr 8, 2007 at 12:29 PM
Subject: Re: the wall and stuff
To: Luke Wolcott

Hello friend,

...I agree whole-heartedly with your idea that maybe we are both
ascending one hill and descending another at the same time...
Maybe it's a cheerful nihilism, and I know I'm an idealist and maybe
someday I'll crash, but I think all of this figuring out has to
happen in its two simplest parts simultaneously - the speculating
and the doing. You need to do the lab with the scratchwork, and I
like that perhaps these two parts have to face in two different directions,
as you would say. The idea of an "answer" doesn't satisfy me really,
because answers aren't real. Sure, they're practical, but they're
not It. In all of these absurd gestures, I'm trying to make
something else matter. What I like about thinking of my hypothetical
urban wind trajectories, for example, is that the maps ignore an
order of importance. Yeah, there are a million variables that
effect the wind, but what happens if I arbitrarily isolate a
couple and grant someone's whisper the same power as a building's
demolition? Rather than directly disempowering dominant
things, I like giving a voice to small, seemingly negligible
things. It's my small way of kicking down hierarchy. So
here we are at the other end of the spectrum, not keying in
on one variable at a time but trying to effectively include
Everything. Yikes. To assume a definitive answer to this
is to assume quite a command of Reality - whatever that is!
Perhaps the project started as a response to a certain

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discontentment with assumed reality, assumed limitations,
assumed boundaries and separateness, and a desire to enter an occult
alternative into the realm of the reasonable. While stubborn and absurd,
I do consider myself a practical girl, a poetical pragmatist, if you
will. But, oh dear, the closer I get to Reality, the further I am from
being sure of anything at all. Now the project is a playful crisis, really.
So, we ask, why bother? What else are we to do - wait around for a
more agreeable reality?...

"We have no idea how long we'd have to continue until we have a chance
of even knowing what our chances are." Yep, that sounds about right.
Except, I want to make clear that there exists a chance. It's
important to me that I believe in these things.

yours,
Liz
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