Mathematical Mysteries

Summer Stretch 2011

Luke Wolcott

Course Content

The starting points for the course will be the following mathematical mysteries:

- Super-fast mental arithmetic tricks, from India, Ukraine, England, and Russia. For example, you'll learn how to quickly multiply 4-digit numbers in your head, figure out the day of the week of any date in history, verify if an addition or multiplication problem is correct, or calculate a square root using only one line of scratchwork. We'll discover together how and why these techniques work.

- Ancient mathematics from cultures around the world - e.g. the Romans, Mayas, Aztecs, and Babylonians. The Egyptians wrote all their fractions as sums of fractions of the form $1/n$. For 17 centuries, the world used a different long division algorithm than the one you learned in school, called Galley Division. Like archaeologists trying to decipher the strange symbols left by these cultures, we'll explore this ancient math, and reinterpret it with our modern tools.

Within each of these examples, there are seeds of deep mathematical ideas. I'll help you to extract these seeds and together we'll develop them a little. This will lead us to new, modern mathematical mysteries.

- Some of the unconventional arithmetic techniques rely on algebra, that in modern times manifests as questions about Diophantine equations (like Fermat's Last Theorem).

- Other super-fast arithmetic relies on modular mathematics, which is a good starting point for understanding modern group theory (and such mysteries as the Rubik's cube!).

- Some ancient mathematics reveals number systems with different bases. This idea is fundamental to how computers work, and we'll look at binary logic, quantum logic, and fuzzy logic.

- Egyptian fractions suggest many questions, about number theory, and sequences and series. There are many modern unsolved problems in these areas, all very accessible. Following this thread, we'll also meet the fascinating mathematicians Paul Erdös and Srinivasa Ramanujan.

- Galley division leads to polynomial division and the wonderful roots of unity. From here, it's not far to Galois theory - one of the most beautiful parts of mathematics ever invented. (And it was invented by a 20 year-old!) For better or for worse, Galois theory is used in modern times as the basis for a lot of cryptography, which is full of mystery.

Course Organization and Materials

I'm not sure we'll get to all of these modern developments - that's a lot of math! Together as a class we'll decide which directions to pursue. Overall, the course organization - what we learn and how we learn it - will be very flexible and collaborative.

Doing research math is a lot like being a scientist - you look for patterns, and suggest reasons for those patterns. This course will be very hands-on and discovery-based. In many cases, before we can figure out the answer, we'll have to figure out what question to ask. This kind of math takes a lot of curiosity and imagination, and is a thousand times more fun than sitting still and being lectured to. So each day you can expect a balance between talks on new material, group work, and independent work. I'll try to surprise you often, with relevant movies, field trips, and games.

What should you bring to class? A notebook, pen/pencil, an open mind, and maybe a calculator.