What's so interesting about Algebraic Topology?

a short answer

In a straightforward way, I think algebraic topology is interesting because the objects it studies are interesting. It is interesting to investigate our human notion of space. Most of the spaces and objects we study have no correlation to anything in physical reality, and this has kept algebraic topology one of the most out-there and least practical fields of math.

In a less direct way, algebraic topology is interesting because of the way we have chosen to study space. By focusing on the global properties of spaces, the developments and constructions in algebraic topology have been very general and abstract. The questions we ask are about large-scale structure, and the aesthetic of the field - what we consider "deep" or "elegant" results - centers on this global perspective. This appeals to me.

At the same time, I find algebra interesting. Algebraic thought is different than geometric, analytic, or topological thought, which all use spacial or visual intuition. Algebra relies solely on relational intuition - our ability to perceive relationships and connections between objects. Such an algebraic intuition can, and must, be developed over time, and awareness of this process fascinates me.

The question "What is interesting about algebraic topology?", I would argue, is equivalent to the question "What is interesting about algebraic topologists?" It is our personalities, perspectives, and aesthetics that have made the field what it is now. In general, I find algebraic topologists very interesting people. In my experience, they are often big-picture thinkers, trying to get a glimpse of the whole forest. They are often more quirky and idealistic, and less grounded than other mathematicians (which is saying something). Presumably this feedback loop reinforces itself - algebraic topology is abstract and useless, and draws to it sympathetic thinkers who will then make it more so. Finally, in my experience, algebraic topologists tend to be good communicators of mathematics. My theory is that, because of the incredibly abstract nature of our field, there is a need to understand concepts on several different levels of generality, often simultaneously, and to move quickly between these levels. Such a multi-level understanding is also required to communicate mathematics successfully. This has also drawn me to the field.

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