

# What are Differential Equations?

*a short answer*

The study of differential equations is one of the most useful and applicable areas of mathematics. An equation is a mathematical statement that shows a relationship between various different quantities. For example,  $E = mc^2$  gives a relationship between energy ( $E$ ) and mass ( $m$ ), and says something very powerful about these physical quantities. A *differential equation* relates a quantity to its derivatives.

What's a derivative? Basically, the derivative of a quantity is the instantaneous rate of change of that quantity. For example, velocity is the derivative of position, and acceleration is the derivative of velocity.

Often, differential equations come from laws of physics. For example, imagine a block hanging from a spring attached to the ceiling. If you give the block a push, it moves according to certain laws. These laws can be described mathematically, and that description is a differential equation. What we'd like is a solution to the differential equation, which means a mathematical formula describing the position of the block in the future. We start with initial conditions and the laws of physics, and by solving a differential equation we obtain complete knowledge of how the block will move.

The power of differential equations comes from the fact that the same differential equation appears in many different physical situation. Whether it's a block on a spring, or a pendulum swinging, or a buoy on the ocean, or a certain simple electrical circuit, the differential equation is the same, and so we can use the same technique to find a solution.

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